# D I S S E R T A T I O N  P R O P O S A L

## Vrishabh Patil

## "Polyhedral Geometry and Algorithmic Methods in Optimization"

Thursday, October 30, 2025
12:00pm
Tepper 4219

Chapter 1 studies the effect of estimation errors in portfolio optimization. We consider robust formulations of the Markowitz model under ellipsoidal uncertainty in expected asset returns. By analyzing diagonal estimation-error matrices, we identify parameter choices that yield arbitrarily small losses in expected return relative to the optimal portfolio. We also provide a data-driven heuristic for selecting the structure and size of the uncertainty set and show empirically that robust models with appropriately chosen diagonal structures consistently outperform the classical Markowitz formulation.

Chapters 2 and 3 focus on the Chvátal rank, defined as the minimum number of rounds of Chvátal cuts required to obtain the integer hull of a rational polyhedron. In Chapter 2, we study the rank of integer-free polyhedra, that is, polyhedra that contain no integer points. For polytopes contained in the unit hypercube $[0,1]^n$, it is known that the Chvátal rank is bounded above and below by n. We show that this linear bound extends to polyhedral in $\mathbb{R}^n$ of dimension at most one but fails for higher dimensions. In particular, we construct a 2-dimensional integer-free polytope whose Chvátal rank is at least three, and show that this bound is tight in dimension two, thereby demonstrating that the linear bound cannot be generalized beyond the 1-dimensional case. In Chapter 3, we show that one can obtain a binary compact extended formulation whose rank is polynomial in the input size of the original polyhedron, in contrast to the exponential rank possible in the original space.

Chapter 4 investigates intersection cuts as a unifying geometric framework for classical cutting-plane procedures in integer programming. We show that split, Chvátal, lift-and-project, and Dantzig cuts can all be viewed as intersection cuts derived from appropriately chosen lattice-free closed, convex sets, revealing a common geometric foundation. Furthermore, we demonstrate that the procedures of Lovász–Schrijver, Sherali–Adams, and Lasserre can be interpreted within this intersection-cut framework. We also analyze the property of strong homogeneity, which requires that closures commute with restriction to faces, identifying which procedures satisfy this property and providing counterexamples for those that do not. This chapter is a work in progress, as we continue to explore the polyhedral geometry of various cutting plane procedures.

Finally, Chapter 5 concerns the computation of nonnegative dyadic solutions to linear systems, motivated by their exact binary representations and natural appearance in combinatorial optimization. Although a polynomial-time algorithm for this problem has been recently established, its computational behavior has not been examined. We implement the algorithm of Abdi et al. (2024) and find that it often produces dense solutions with large denominators. To address this, we propose a new method that combines column generation with a structured null-space search to obtain sparse dyadic solutions with small denominators. Preliminary experiments on randomized and benchmark instances demonstrate promising performance. This work is ongoing, and we are extending our experiments to build a comprehensive computational study of the proposed algorithm.

Together, these chapters advance both the theory and computation of optimization. They contribute new insights into the rank and structure of cutting plane procedures, introduce unifying geometric perspectives, and develop practical algorithms for structured linear systems.

Proposed Committee: Gérard Cornuéjols (Chair), Willem-Jan Van-Hoeve, Jeffrey Linderoth, and Santanu S. Dey